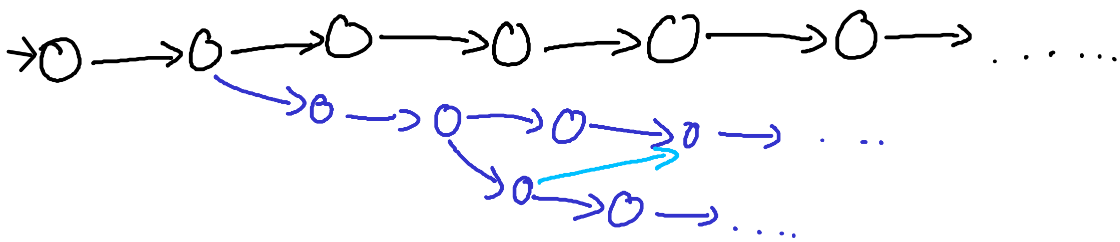
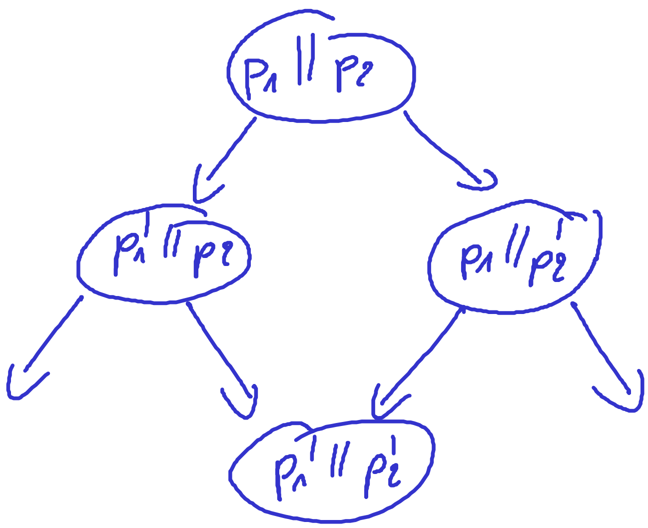
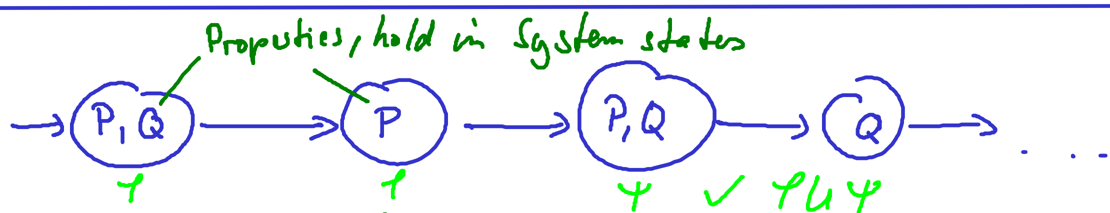
Linear Time Logic (LTL):







$X(P\,U\,Q)$

$(XP)\,U\,Q$

Syntax of LTL:

$P \in Prop \Rightarrow P \in LTL$, $true, false \in LTL$

$\varphi, \psi \in LTL \Rightarrow \neg\varphi,\ \varphi \vee \psi,\ \varphi \wedge \psi \in LTL$

$\Rightarrow \underbrace{X\varphi}_{\text{Next}},\ \underbrace{\varphi\, U\, \psi}_{\text{Until}} \in LTL$

Abreviations:

$F\varphi := true\ U\ \varphi$ Finally — liveness

$G\varphi := \neg F \neg\varphi$ Globally — safety

$F^\infty\varphi := G\,F\varphi$ Infinitely often φ holds — for fairness often as preconditions

$G^\infty\varphi := F\,G\varphi$ φ hold always wit finitely many exceptions

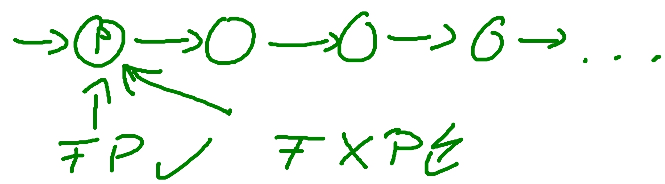
Semantics:

It's defined for an infinite path $\pi = p_0\, p_1\, p_2 \ldots$ with $p_i \in \mathcal{P}(Prop)$ $\left(\pi \in \mathcal{P}(Prop)^\omega\right)$.

- $p_0\pi \models P$ iff $P \in p_0$ $\qquad p_0\pi \models \neg P$ iff $P \notin p_0$
- $\pi \models \varphi \wedge \psi$ iff $\pi \models \varphi$ and $\pi \models \psi$
- $\pi \models \varphi \vee \psi$ iff $\pi \models \varphi$ or $\pi \models \psi$
- $\pi \models \neg \varphi$ iff $\pi \not\models \varphi$
- $p_0\pi \models X\varphi$ iff $\pi \models \varphi$
- $\pi \models \varphi\, U\, \psi$ iff $\exists i \in \mathbb{N} : \forall j < i : p_j p_{j+1} \ldots \models \varphi$ and $p_i p_{i+1} \ldots \models \psi$
- $\pi \models F\varphi$ iff $\exists i \in \mathbb{N} : p_i p_{i+1} \ldots \models \varphi$
- $\pi \models G\varphi$ iff $\forall i \in \mathbb{N} : p_i p_{i+1} \ldots \models \varphi$

Equivalences on LTL:

FXP



$$FX\varphi \approx XF\varphi$$

use laws to push negation inside

$$\neg(\varphi \wedge \psi) \approx \neg\varphi \vee \neg\psi$$
$$\neg(\varphi \vee \psi) \approx \neg\varphi \wedge \neg\psi$$
$$\neg(\neg\varphi) \approx \varphi$$
$$\neg X\varphi \approx X(\neg\varphi)$$
$$\neg F\varphi \approx G\neg\varphi$$
$$\neg G\varphi \approx F\neg\varphi$$
$$\neg(\varphi\, U\, \psi) \approx (\neg\varphi)\, R\, (\neg\psi)$$

Release operation, strange semantics but useful for negating Until.

$$F\varphi \approx \varphi \vee XF\varphi$$
$$G\varphi \approx \varphi \wedge XG\varphi$$
$$\varphi\, U\, \psi \approx \psi \vee (\varphi \wedge X(\varphi\, U\, \psi))$$
$$\varphi\, R\, \psi \approx \psi \wedge (\varphi \vee X(\varphi\, R\, \psi))$$

enrole the fixed point to check for the initial state of path and to shift the remaining decision to the path starting in the next state.